# Technical Comments

## Comments on "Structural Optimization in the Dynamics Response Regime: A Computational Approach"

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IN Ref. 1 the authors use the method of feasible directions in the optimization part of their paper. Although the usable feasible direction is correctly defined in their Eqs. (27) and (28), their method of obtaining the optimum feasible direction is not optimum.

In order to find an optimum feasible vector the authors of Ref. 1 solve a linear programming problem in each interaction. This approach is too long and expensive for practical applications. Recently[2] a simpler and more direct method has been developed for finding the optimum feasible direction. This method involves operations on vectors by sweeping out of the direction vector the components which lead to violation of the active constraints.

Furthermore, the method presented in Ref. 2, and overlooked in Ref. 1, leads to a faster convergence towards the optimum and avoids certain local optimums. A generally "steeper" direction vector is used because only the effective-active constraints are included in the sweeping process.

If the component of the direction vector along the gradient of an active constraint is positive, the movement in that direction will not lead to a violation of the active constraint. Disregarding such active constraints results in the steepest feasible direction, i.e., the direction vector closest to the gradient of the objective function.

In conclusion, incorporating the approach presented in Ref. 2 for finding the optimum feasible direction will lead to an improved optimization technique, and further the contribution of Ref. 1.

### References

[1] Fox, R. L. and Kapoor, M. P., "Structural Optimization in the Dynamics Response Regime: A Computational Approach," *AIAA Journal*, Vol. 8, No. 10, Oct. 1970, pp. 1798–1804.

[2] Ridha, R. A. and Wright, R. N. "Minimum Cost Design of Frames," *Journal of the Structural Division*, ASCE, Vol. 93, No. ST4, Paper 5394, Aug. 1967, pp. 165–183.

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## Reply by Author to R. A. Ridha

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RIDHA has overlooked several points in his comment. To begin with, the issue of efficiency of the direction finding solution itself is academic because the linear program which is solved at each iteration of Zontendijk's method of feasible directions is a mere pip of calculation when compared with the other arithmetic involved in solving a "practical" problem. A second, and perhaps more important point, is the fact that the "new" method described by Ridha and Wright is exactly Rosens gradient projection method. This method was first described in Ref. 1 although the discussion and comment in Sec. 4.3 of Ref. 2 may be helpful. The method was also used to solve the problem in Ref. 3. Although it is somewhat obscured by the form of the calculation, Ridha and Wright do a Gauss elimination process to perform the projection.

Rosens method is indeed a superb technique but its main usefulness is limited to problems with linear constraints. For problems with convex constraint surfaces (as are typical in many applications) the direction produced by it is infeasible and the additional costly step to get back into the feasible domain as described by Ridha and Wright is necessary. Incidentally, with the "pushoff factors" in the method of feasible directions set to zero, the method produces the projected gradient vector and it is therefore a sub-method of the feasible direction method. In conclusion, the method was not "overlooked" but disregarded as inappropriate for this class of problems.

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### References

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2. Fox, R. L., "Optimization Methods for Engineering Design," Addison-Wesley, Reading, Mass., 1971, pp. 196–205.
3. Brown, D. M. and Ang, A. H. S., "Structural Optimization by Nonlinear Programming," *Journal of the Structural Division, Proceedings of the American Society of Civil Engineers*, Vol. 92, No. ST6, Oct. 1966, pp. 319–340.

## Comment on "Stability of a Spinning Body Containing Elastic Parts via Liapunov's Direct Method"

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IN his paper[1] L. Meirovitch has solved the problem on the stability of a spinning body containing elastic parts. He has obtained the system of the differential equations by means of Hamilton's principle. The motion of the system is described by a "hybrid" set of ordinary differential equations and partial differential equations. Considering the Hamiltonian as a Liapunov function and functional simultaneously the author has obtained the sufficient conditions of the stability.

The author asserts, that he has presented "a new method of approach to the stability problem of hybrid systems . . . The method consists of an extension of the Liapunov direct method by considering for testing purposes a hybrid form, that is a form which is a Liapunov function and functional

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at the same time. The method works directly with the hybrid set of differential equations . . . ."

The assertion, that the method is the new one, is not quite right. The method has been developed first in papers[2,3] (see also the monograph[4]) for applying to the problem on stability of solid bodies with liquid-filled cavities. The hybrid set of the differential equations of the system motion has been also obtained with the help of Hamilton's principle. The energy or a combination of the energy and the first integrals of the hybrid set of differential equations has been considered as Liapunov function and functional simultaneously.

This method is indeed the general and rigorous one and should be applicable to the stability analysis in many areas.

**References**

[1] Meirovitch, L., "Stability of a Spinning Body Containing Elastic Parts via Liapunov's Direct Method," *AIAA Journal*, Vol. 8, No. 7, July 1970, pp. 1193–1200.

[2] Rumyantsev, V. V., "On the Stability of Rotational Motions of a Rigid Liquid-Filled Body," *PMM*, Vol. 23, Dec. 1959.

[3] Rumyantsev, V. V., "Stability of Motion of Solid Bodies with Liquid-Filled Cavities by Lyapunov's Methods," *Advances in Applied Mechanics*, Vol. 8, Academic Press, New York, 1964, pp. 183–232.

[4] Moiseyev, N. N. and Rumyantsev, V. V., "Dynamic Stability of Bodies Containing Fluid," Springer-Verlag, New York, 1968.

## Reply by Author to V. V. Rumyantsev

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IN his Comment, Rumyantsev disputes the assertion made in Ref. 1 that "A new method of approach to the stability problem of hybrid systems, . . . , is presented." He argues the point by contending that the "general and rigorous" method was developed in Ref. 2 (Ref. 3 presents essentially the same information as Ref. 2). This contention, however, does not find much support in facts, and, indeed, a close examination of Ref. 1 reveals very little resemblance to Ref. 2. Although both works are concerned with hybrid systems and consider stability analyses based on Liapunov's direct method, there the similarities end. Whereas the mathematical model of Ref. 1 is a solid body that is part rigid and part elastic, Ref. 2 considers rigid bodies with fluid-filled cavities. The real difference, however, lies not so much in the mathematical model, or the problem formulation, but in the method of approach to the stability problem. Indeed, Ref. 2 uses the standard Liapunov method to test the stability of a discrete system, whereas Ref. 1 develops a technique, based on the Liapunov direct method, to test the stability of a hybrid system. To be specific, Ref. 2 reduces the hybrid system to a discrete one by either considering cavities entirely filled with an ideal fluid and assuming that " . . . the motion of the fluid is completely defined by a finite number of variables" or by considering cavities partially or completely filled by an ideal or viscous fluid and assuming that " . . . in this case it is also possible to state the stability problem with respect to a finite number of variables by introducing certain quantities that integrally describe the motion of the fluid." To analyze such systems, Rumyantsev presents "Two theorems on stability with respect to a part of the variables, which can be regarded as modifications of the Lyapunov stability theorem." In conclusion, Rumyantsev interprets stability in a finite dimensional vector space consisting of the rigid body motion and a finite number of variables (depending on time but not on space) representing the fluid, thus avoiding many of the difficulties inherent in a stability analysis of truly hybrid systems.

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The method of Ref. 1, by contrast, does not resort to any discretization scheme and interprets stability " . . . in a space $S$ which can be regarded as the cartesian product of the finite dimensional vector space and the function space." The vector space is associated with the "rigid-body motion" and the function space with the motion of the elastic continuum. Since the system is hybrid, an expression which is both a function and $l$ at the same time is considered for testing purposes; the expression is the system Hamiltonian. Difficulties caused by terms involving partial derivatives with respect to spatial variables in the Hamiltonian are circumvented by invoking certain properties of Rayleigh's quotient and devising a new testing function $\kappa$ which is known to be smaller in value than the Hamiltonian. Moreover, defining a testing density function $\hat{\kappa}$ for every point of the elastic domain $D_e$, where $\hat{\kappa} = \kappa/D_e$, the sign properties of $\hat{\kappa}$ are checked at every point of $D_e$.

The author is confident that a more in-depth study will convince Rumyantsev that Ref. 1 does indeed contain many novel ideas not found anywhere else. The application of the techniques developed in Ref. 1 to test the stability of motion of rigid bodies with fluid-filled cavities is in the realm of possibility.

**References**

[1] Meirovitch, L., "Stability of a Spinning Body Containing Elastic Parts via Liapunov's Direct Method," *AIAA Journal*, Vol. 8, No. 7, July 1970, pp. 1193–1200.

[2] Rumyantsev, V. V., "Stability of Motion of Solid Bodies with Liquid-Filled Cavities by Lyapunov's Method," *Advances in Applied Mechanics*, Vol. 8, Academic Press, New York, 1964, pp. 183–232.

[3] Moiseyev, N. N. and Rumyantsev, V. V., *Dynamic Stability of Bodies Containing Fluid*, Springer-Verlag, New York, 1968.

## Comment on "Spectroscopic Study of Ion-Neutral Coupling in Plasma Acceleration"

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MALLIARIS and Libby[1] have apparently overlooked an effect which may contribute appreciable errors to their measurements of axial velocity of neutrals in an MPD flow. Their velocity measuring technique cannot discriminate between particles which emanate from the thruster and identical particles which diffuse into the beam from the background. Both will be excited by collisional processes in the core of the beam and both will contribute light to the spectral line being observed. Their relative contributions will be in proportion to their relative densities. If the density of the background neutrals is not negligible compared to the density of the beam neutrals, a spectroscopic observation will yield a composite line.

The apparent doppler shift will give some sort of weighted average of the velocities of the two types of neutrals. Unless

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